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A-236-01-01

M.Sc. DEGREE EXAMINATION, DEC. 2015
FIRST SEMESTER
Branch : MATHEMATICS

MA 101 : ALGEBRA

(Under CBCS w.e.f. 2015-16)

(Common to supplementary candidates also i,e., who appeared in Nov. 2014 and earlier)

(Common to Non-CBCS)

Time : 3 Hours

Max. Marks : 90

SECTION - A

Answer any **FOUR** questions. **All** questions carry **equal** marks

(Marks : 4×4½=18)

1. Let G be a group. Show that G is a G-set with respect to the group action '*' defined by $a*x=axa^{-1}$, for all $a \in G$ and $x \in G$

2. State and prove Cayley's theorem.

3. Let $f : R \to S$ be a homomorphism of a ring R into a ring S. Then prove that ker f = $\{0\}$ if and only if f is one-one.

4. If R is a ring with unity then show that each maximal ideal is prime.

5. Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime.

6. Show that the ring of Gaussian integers $R = \{m + n\sqrt{-1}, m, n \in Z\}$ is a Euclidean domain.

7. State and prove schur's lemma.

8. If M is finitely generated free module over a commutative ring R, then prove that all bases of M are finite.

UNIT - III

13. Every Principal ideal domain is UFD, but a UFD is not necessarily a Principal ideal domain.

(Or)

14. Let R be a UFD. Then the polynomial ring R[x] over R is also a UFD.

UNIT - IV

15. Let R be a ring with unity and M be an R-module then the following statements are equivalent:

    i) M is simple

    ii) $M \neq 0$ and M is generated by any $X \in M$ where $x \neq 0$.

    iii) $M \cong \frac{R}{I}$ where I is a maximal ideal of R.

(Or)

16. Let V be a vector space over a field F with basis $(e_i), i \in A$, then prove that

    i) $V = \oplus \sum_{i \in \supseteq \Lambda} Fe_i \cong \oplus \sum_{i \in \supseteq \Lambda} F_i = F$

    ii) V is completely reducible.

    iii) If W is a subspace of V, then a sub space W', such that $V = W \oplus W'$

    iv) Suppose $|A| < \infty$ and let $\{e_1, e_2, ... e_k\}$ be a linear independent subset of V. Show that there exists a basis of V containing $\{e_1, e_2, ... e_k\}$

[Total No. of Pages : 3 **A-236-01-05**

M.Sc. DEGREE EXAMINATION, DEC. 2015
FIRST SEMESTER
BRANCH : MATHEMATICS
MA 105 : COMPLEX ANALYSIS
(Under CBCS w.e.f. 2015-16)
(Common to supplementary candidates also i.e, who appeared in Nov. 2014 and earlier)
(Common to Non-CBCS)
(Common to Applied maths)

Time : 3 Hours Max. Marks : 90

PART - A

Answer any FOUR questions. All questions carry equal marks.

(Marks 4×4½=18)

1. Does the function

   $f(z) = e^x(\cos y + i\sin y)$ satisfy Cauchy - Riemann equations?

2. Find the rotation magnification and (finite) fixed point, if it exists for the following transformation and write it in canonical form $w - z_0 = \alpha(z - z_0), w = 2z + 1 - 3i$

3. Show that every Mobius transformation $W = L(Z) = \dfrac{az+b}{cz+d}(ad - bc \neq 0)$ is circle-preserving.

4. Find the point symmetric to the point 2+i with respect to the circle $|z| = 1$

5. Evaluate the integral $\int_L |Z| dz$ along the circle $|Z| = R$

6. State and prove Morera's theorem.

7. Find the radius of convergence of the power series $\sum_1^\infty n^n z^n$

8. State and prove Liouville's theorem.

b) Evaluate $\int_c z^{-2} dz$ around the circle $|z-1| = 1$.

UNIT - IV

15. a) Let K be the disk $|z-a| < R$ and suppose f(z) is analytic in K. then prove that f(z) has Taylor expansion in K with $f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n$.

b) Find the Taylors expansion of cos z at $z = \frac{\pi}{4}$.

(Or)

16. a) State and prove Liouvilles theorem.

b) State and prove uniqueness theorem analytic functions :Let f(z) and g(z) be two functions analytic in the same domain G, and suppose f(z) and g(z) coincide at all points of a subset E of G with a limit point $z_0 \in G$. Then f(z) and g(z) coincide in the whole domain G.

M.Sc.DEGREE EXAMINATION, DECEMBER, 2015

FIRST SEMESTER

Branch: 1(A) Mathematics

MA 104- COMPUTER ORIENTED NUMERICAL METHODS

(Revised Regulations CBCS from 2014-2015)

(Common for both CBCS and Non-CBCS)

(Common to M.Sc. Appl, maths)

Time : 3 Hours Max. Marks : 90

PART - A

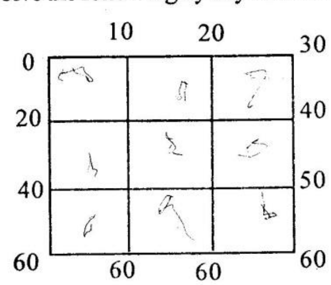
Answer any **FOUR** of the following. Each question carries 4 ½ marks.

(Marks: 4×4½=18)

1. Find $y\left(\frac{\pi}{6}\right)$ given that (0,0), $\left(\frac{\pi}{2},1\right)$, $(\pi,0)$ satisfies the function $y = \sin x\,(0 \le x \le \pi)$ using cubic spline approximation

2. For the D.E $y' = x - y^2$ with $y(0) = 1$ find $y(0.1)$ using Taylors series.

3. Solve the following by any known method.



4. Write down the finite difference analogues $\frac{\partial^2 u}{\partial x \partial y}$ and $\frac{\partial^2 u}{\partial y \partial x}$

5. What are the various Data types and explain them.

6. Write C assignment statements to evaluate the following.

Side = $\sqrt{a^2+b^2-2ab\cos(x)}$

7. Write a program to write x^n Using WHILE loop.

8. Write a program using FOR LOOP to Print the squares of 20 given numbers both positive and negative.

PART - B

Answer **ONE** Question from each Unit. Each question carries 18 marks.

(Marks: 4×18=72)

UNIT - I

9. Given $\frac{dy}{dx}=y-x$ where $y(0)=2$ find $y(0.1)$ and $y(0.2)$ correct to four decimal places using Runge Kutta 4th order method.

(Or)

10. Derive Milnes method.

UNIT - II

11. Write down the finite difference analogues of the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ and solve it for the region bounded by the square $0 \le x \le 4$ and $0 \le y \le 4$ the boundary conditions being

$U=0$ at $x=0$ and $u=8+2y$ at $x=4$

$U=\frac{1}{2}x^2$ at $y=0$ and $u=x^2$ when $y=4$

(Or)

12. Solve the boundary value problem $u_{tt}=4u_{xx}$ Subject to the conditions $U(0,t)=0=u(4,t)$, $u_t(x,0)=0$, $u(x,0)=4x-x^2$ with $h=1$, $\alpha=1$ and $k=0.5$.

UNIT - III

13. Write a program to evaluate the quadratic equation $ax^2+bx+c=0$. 8

(Or)

14. Write a program to count the number of boys whose weight is less than 5- kgs and height is greater than 170cms assuming suitable data.

UNIT - IV

15. Write a program to write binomial coefficients for any set of m and x.

(Or)

16. Write a program to compute and print a multiplication table for the numbers 1 to 5 as shown below using two dimensional array.

	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	.	.	.
4	4	8	.	.	.
5	5	10	.	.	25

A-236-01-03

M.Sc. DEGREE EXAMINATION, DEC. 2015
FIRST SEMESTER
Branch : MATHEMATICS

MA 103 : ORDINARY DIFFERENTIAL EQUATIONS

(Under CBCS w.e.f. 2015-16)

(Common to supplementary candidates also i.e., who appeared in Nov. 2014 and earlier)

(Common to Non-CBCS)

(Common to Appl. Maths)

Time : 3 Hours   Max. Marks : 90

PART-A

Answer any FOUR questions. All questions carry equal marks.

(Marks 4×4½=18)

1. Let u(x) be any non-trivial solution of $U'' + q(x)u = 0$, where q(x)>0 for all x>0. If $\int_1^{\infty} q(x)dx = \infty$, then prove that u(x) has infinitely many zeros on the positive x-axis

2. Find the eigen values λ_n and eigen functions $y_n(x)$ for the equation $y'' + \lambda y = 0$, where y(0)=0 $y(\pi/2) = 0$

3. For the differential equation $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$, locate and classify its singular points on the x-axis.

4. Determine the nature of the point x=0 for the equation $x^2y'' + (\sin x)y = 0$

5. Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$

6. Show that $2^\wedge\Gamma(n+\frac{1}{2}) = 1.3.5.......(2n-1)\sqrt{\pi}$

7. Using Picard's method of successive approximation solve $y' = x + y,\ y(0) = 1$

8. Show that $f(x,y)=xy^2$ satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$

PART-B

Answer **all** questions. Each question carries **equal** marks.

(Marks:4×18=72)

9. a) Find the normal form of Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ and use it to show that every non-trivial solution has infinitely many positive zeros.

(OR)

b) State and prove Sturm comparison theorem.

10. a) The equation $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$ has only one frobenius series solution. Find the general solution.

(OR)

b) Find the general solution of $(1-e^x)y'' - \frac{1}{2}y' - e^x y = 0$ near the singular point x=0 by changing the independent variable to $t=e^x$

11. a) State and prove orthogonal properties of the Legendre polynomials

(OR)

b) i) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ and $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$

ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

12. a) State and prove Picard's existence theorem.

(OR)

b) Let f(x,y) be a continuous function that satisfies a Lipschitz condition $|f(x,y_1) - f(x,y_2)| \le K|y_1 - y_2|$ on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0,y_0) is any point of the ship. then prove that the IVP $y' = f(x,y),\ y(x_0) = y_0$ has one and only one solution y=y(x) on the interval $a \le x \le b$

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A-236-01-02

M.Sc. DEGREE EXAMINATION, DEC. 2015

FIRST SEMESTER

Branch : MATHEMATICS

MA 102 : REAL ANALYSIS

(Under CBCS Revised syllabus w.e.f. 2015-16)

(Common to Non-CBCS)

(Common to Applied Mathematics)

Time : 3 Hours

Max. Marks : 90

PART-A

Answer any FOUR questions. All questions carry equal marks.

(Marks 4×4½=18)

1. If p^* is a refinement of p, then prove that $L(p, f, \alpha) \leq L(p^*, f, \alpha)$

2. State and prove fundamental theorem of calculus.

3. Show that the sequence of functions $\{f_n\}$ defined on E converges uniformly on E if and only if for every $\varepsilon > 0$ There exists an integer N such that

   $n \geq N,\ m \geq N,\ x \in E$ *implies* $|f_n(x) - f_m(x)| \leq \varepsilon$

4. If K is compact if $f_n \in \mathscr{C}(K)$ for n=1,2,3,... and if $\{f_n\}$ is point wise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

5. Test the convergence of $\int_0^{\pi/2} \frac{\sin x}{x^P} dx$

6. Examine the convergence of $\int_0^{\infty} \frac{x^2\, dx}{\sqrt{x^5 + 1}}$

7. If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ & (x,y) = (0,0) \end{cases}$

Show that both the partial derivatives exist at (0,0) but the function is not continuous there of.

8. Find maxima and minima of the function $f(x,y) = x^3 - y^3 - 3x - 12y + 20$

PART-B

Answer all questions.All question carry equal marks.

(Marks:4×18=72)

9. a) Show that $f \in \Re_{\circ}(\alpha)$ on [a,b] if and only if for every $\varepsilon > 0$ there exists a portion P such that $U(p,d,\alpha) - L(p,f,\alpha) < \varepsilon$

b) Suppose $f \in \Re(\alpha)$ on [a,b] $m \leq f \leq M$, ϕ is continuous on [m,M] and $h(x) \equiv \phi(f(x))$ on $[a,b]$ Then show that $h \in \Re(\alpha)$ on $[a,b]$

(OR)

10. a) If f is monotonic on [a,b] and if α is continuous on [a,b], then prove that $f \in R(\alpha)$

b) If r' is continuous on [a,b] then prove that r is rectifiable and $\wedge(r) = \int_a^b |r'(t)| dt$

11. a) Suppose $\{f_n\}$ is a sequence of functions differentiable on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If $\{f_n'\}$ converges uniformly on [a,b] then prove that the sequence $\{f_n\}$ converges uniformly on [a,b] to a function f and $f'(x) = \lim_{n \to \infty} f_n'(x)$ $(a \leq x \leq b)$

b) Show that there exists a real continuous function on a real line which is nowhere differentiable

(OR)

12. State and prove stone weirstress theorem.